

BG Research Online

Compton, A. and Boylan, M. (2023) *From flexible to restricted mathematics: 15 years of Ofsted mathematics reports.* Mathematics Teaching, 289. pp. 31-36. ISSN 0025-5785

This is an Accepted Manuscript published by *Association of Teachers of Mathematics* in its final form on 8th December 2023 at: <u>https://atm.org.uk/Mathematics-Teaching-Journal-</u><u>Archive</u>

This version may differ slightly from the final published version.

Copyright is retained by the author/s and/or other copyright holders.

End users generally may reproduce, display or distribute single copies of content held within BG Research Online, in any format or medium, for <u>personal research & study</u> or for <u>educational or other</u> <u>not-for-profit purposes</u> provided that:

- The full bibliographic details and a hyperlink to (or the URL of) the item's record in BG Research Online are clearly displayed;
- No part of the content or metadata is further copied, reproduced, distributed, displayed or published, in any format or medium;
- The content and/or metadata is not used for commercial purposes;
- The content is not altered or adapted without written permission from the rights owner/s, unless expressly permitted by licence.

For enquiries about BG Research Online email <u>bgro@bishopg.ac.uk</u>.

From flexible to restricted mathematics: 15 years of Ofsted mathematics reports

Ashley Compton^a and Mark Boylan^b ^a Bishop Grosseteste University

^b Sheffield Hallam University

Introduction

In the context of high-stakes accountability, The Office for Standards in Education (Ofsted), the school inspection service, has considerable influence over mathematics teaching. It does this through inspection but also through reports and guidance for teachers and school leaders.

Members of the ATM know this. There were responses to the 2021 Ofsted research review in MT278 raising concerns and the ATM/MA Primary Working Group (2021) published guidance for early years and primary, cautioning the need for careful interpretation of Ofsted's claims. In 2023, as a complement to the 2021 research review, Ofsted published a subject report comparing current practice to review recommendations. In this article, using the theme of same and different, we consider both recent Ofsted reports, comparing them with those published in 2008-2012.

Our framing for the comparison is that 15 years ago there was a broad consensus across mathematics education in policy-oriented and funded bodies such as Ofsted, QCA, the NCETM, and the National Strategies, as well as independent organisations, such as subject associations and the Advisory Committee on Mathematics Education. We use 'broad', to qualify consensus, because all organisations did not agree on everything. However, there was general agreement on key issues. Partly, this was possible because the overall paradigm of mathematics education was one of flexibility.

By paradigm we refer to a 'world view' of interconnected beliefs about mathematics, teachers, and learners. The concept is a looser and more composite way of talking about an overall view of mathematics than Ernest's idea of ideologies of mathematics education (Ernest, 1998). In this article, we focus on problem solving, calculations, and classroom talk as exemplifying different paradigms, to give insight into how teachers, learners, and mathematics are viewed.

Considering the paradigm that was supported in England in the 2000s, 'flexible' conveys that there are no hard and fast rules about how to teach and learn mathematics applicable to all content, all learners, and all ages. In short, one size doesn't fit all. Other characteristics of this paradigm were pragmatism and inclusivity about teaching approaches with a willingness to see different views and perspectives as legitimate. In Ofsted's reports from this period, flexibility was also a desirable characteristic of learners doing mathematics.

This paradigm was informed by the practical knowledge of teachers, curriculum developers, and CPD leaders, supported by research evidence. An important synthesis of that evidence base is found in the Nuffield-funded 'Key Understandings in Mathematics Education' (Nunes et al, 2008). This flexible

paradigm was not found in all schools, but Ofsted and other organisations were trying to encourage change towards this.

Ofsted's view has changed. We call Ofsted's current paradigm 'restricted' because it restricts what mathematics teaching and mathematics are and, more importantly, restricts learners' experience of mathematics. Interestingly, Ofsted in 2012 used the term restricted to describe practices that Ofsted advocated in 2023.

Comparing Ofsted then and now

This article is based on a larger analysis of mathematics education policy texts, including those of Ofsted. Here, we focus on five Ofsted texts (Table 1). To help readers avoid confusion, rather than using normal citation conventions we refer to them by the date and paper initials. The different evidential bases are noted. The research review (2021) represented a repositioning of Ofsted as a research broker, selecting literature to support their guidance before collecting evidence of its application (CMS, 2023). Previously Ofsted collected evidence, then derived guidance from this. See Table 1.

Era	Year	Title	Purpose	Evidence base	Sample	Reference
THEN	2008	Understanding the Score	Report on practice	Inspections across a range of schools (plus sample of effective schools)	84 (6)* primary 108 (1)* secondary	UtS
	2011	Good Practice in Primary mathematics	Practice guide	Research visits to effective schools	10* maintained and 10* independent primary schools	GPP
	2012	Made to Measure	Report on practice	Inspections across a range of schools (plus sample of effective schools)	160 (11)* primary 160 (2)* secondary	MtM
MON	2021	Ofsted Research Review Series: mathematics	Practice guide	Research review	N/A	RRM
	2023	Coordinating Mathematical Success: Mathematics Subject Report	Report on practice	Inspections across range of schools	25 primary 25 secondary	CMS

Table 1 Ofsted mathematics reports: then and now

We treat the texts 'then' and 'now' as sets, acknowledging, however, that there are differences between texts within sets. For example, RRM (2021) presented approaches as applicable across all age groups, while CSM (2023) notes differences between primary and secondary.

The Ofsted reports cover a wide range of mathematical topics – too many to fully consider here. In our comparison, we focus on problems, calculations, and classroom talk. We discuss these separately, but, in all texts, they are intertwined. To demonstrate this, we use a visual representation showing the intersections between the three foci, modelled on a Venn diagram structure. The way these are viewed across the two time periods exemplifies the paradigmatic differences in relation to teachers, learners, and mathematics.

Problems

Reports in both periods share the recognition that problem solving features too little in schools. However, the place of problems and problem solving in mathematics differs considerably. In the earlier reports, problem solving is both an essential part of mathematics and a means of teaching mathematics, although seen in too few lessons. Good practices in problem solving included using open-ended investigations that provided challenges to pupils, developed their reasoning, and were related to deep conceptual understanding (UtS, 2008; GPP, 2011; MtM, 2012). A common feature identified in 2011 was that 'Pupils' extensive experience of solving problems deepens their understanding and increases their fluency and sense of numbers (GPP, 2011: 20).'

UtS (2008) specifically criticised the practice of giving pupils a 'recipe' for solving problems because it did not help pupils develop their independent reasoning and communication skills. Good practice (GPP, 2011) involved providing pupils with a wide range of problems within mathematics itself, as well as cross-curricular work. Varied, open-ended investigations required pupils to choose how they approached the task and how they would present their solution and the thinking behind it, demonstrating their conceptual understanding (UtS, 2008).

There is a direct contrast with many of the aspects discussed in the earlier reports. Word problems are the main form of problem solving discussed and open-ended problem solving is dismissed as 'might be enjoyable for both teachers and pupils, but it does not necessarily lead to improved results (RRM, 2021: 25).' Conceptual understanding is not foregrounded in RRM and is presented as only the relationship between facts (RRM, 2021: 6). The focus is on 'conditional knowledge' combining facts and methods 'transformed into strategies when pupils learn to match the problem types that they can be used for (RRM, 2021: 6).' The focus is on worked examples, which pupils copy so that they can learn specific strategies to use for different problem types (RRM, 2021; CMS, 2023). Notably, and in contrast, the 2012 report criticised teaching that focused on end of topic word problems which all involved one operation.

Ofsted, now, acknowledges that secondary students will need to determine how to approach problems in exam situations where problems relate to different aspects of mathematics (CMS, 2023). The way to do this is to address weaknesses in fluency with calculations and factual knowledge (CMS, 2023: 17) rather than engaging with problems. Rather than developing fluency through problem solving, as Ofsted

recommended previously, automaticity with number facts is seen as a prerequisite for problem solving (RRM, 2021: 12).

Calculation

The recent reports have greater focus on calculation and less on problem solving than earlier ones, and there is a difference what calculation means. These echo the differences in the meaning of problems in mathematics and are exemplified in the following quotes:

'A key message of this report is that pupils whose understanding of number, its structures and relationships, is developed alongside their proficiency with arithmetic have the grounding so necessary for future learning, particularly of algebra.' (GPP, 2011: 26)

'There is a difference between methods that help pupils understand concepts and perform mental calculations and methods that are efficient and useful now and in the next stage of learning... Pupils should learn the most efficient, systematic, and accurate mathematical methods so that they can use them for more complex calculations and in their next stage of learning.' (CMS, 2023: 15)

Overall, there are three important differences.

1. Understanding, automaticity and facts together versus facts, automaticity and understanding as a sequence

A commonality is the importance of practising mathematics for learning and the fact that fluency and understanding are connected. However, the nature and overall purpose of practising changed. Previously, Ofsted warned against children copying the steps of methods without understanding the rationale because they would not develop their conceptual understanding and struggle to apply the method (UtS, 2008; MtM, 2012). The current Ofsted information processing model of learning emphasises gaining automaticity first, then looking for patterns and connections, and developing mathematical understanding after (RRM, 2021; CMS, 2023). This contrasts with earlier recommendations of how tasks can support practising mathematics alongside developing understanding. Indeed, being able to perform the method without understanding it was seen as holding pupils back (UtS, 2008).

2. Informal and formal methods-good ways or the right way

Both informal and formal methods of calculation were discussed in all the reports, but there was a marked difference between the two eras. In the earlier reports (UtS, 2008; GPP, 2011; MtM, 2012), the inspectors noted that once pupils had learned formal methods, they defaulted to these, even when it was not the most efficient method. For example, using short multiplication for 99 x 8 instead of 100 x 8 – 1 x 8 (GPP, 2011: 24). Inspectors praised schools that encouraged flexibility in calculations because it empowered pupils to choose the most appropriate method. However, this was highlighted as poor practice in 2023 because allowing pupils to choose their method 'could lead to pupils choosing 'easier' methods and not getting enough practise in using methods of most use (CMS, 2023: 15).'

In the primary section, the grid method was described as limited because larger numbers need the formal algorithm (CMS, 2023: 15), although its potential to be extendable to complex situations was noted in the secondary section (CMS, 2023: 29). In contrast, in 2011, Ofsted noted its power in helping children develop understanding of multiplying decimals, expanding algebraic expressions, and factorisation. The implication that there is a single right way to do mathematical calculations also extends to how mathematics is presented. RRM (2021) called for presentation rules to be taught to develop neatness, which, it claims, lead to success. Unusually for the research review, there were no references to any literature in this section. This aspect was picked up in CMS (2023), claiming good presentation was a source of pride for the pupils while poor handwriting held children back in mathematics.

3. Number sense versus number methods

Ofsted's earlier approach to calculation is consistent with aiming to develop number sense. This is exemplified by how place value and models of the structure of the number system are viewed. Place value was described as one of the big ideas in mathematics (UtS, 2008) and considered a prerequisite for understanding formal algorithms (GPP, 2011; MtM, 2012). The 2021 review did not mention place value, and the only reference to it in 2023 suggested that while the grid method of multiplication helped develop understanding of place value, it was 'at the expense of developing automaticity in using efficient and formal methods (CMS, 2023: 15).'

Some aspects were the same in all reports:

- Knowing mathematical facts matters
- Calculation policies are important to establish consistency in teaching, models, and sequences to ensure progression

Classroom talk

All the reports referred to using mathematical language. Earlier reports valued children expressing their reasoning to others, to talk partners, in group collaboration, or to the teacher. Reasoning through discussion features less in the recent reports, although CMS (2023: 19) recommends using the questions like 'What is the same and what's different?' In 2021, discussion is acknowledged only in a recommendation that teachers balance opportunities for discussion with quiet periods for thinking (RRM, 2021: 29).

Teaching mathematical vocabulary was highlighted then and now, recommending teachers model precise use of mathematical language and notation. Previously, dialogue between adults and children was a focus of early years practice, as was capitalising on the mathematics within everyday classroom activities (UtS, 2008; GPP, 2011; MtM, 2012). Songs and rhymes were also mentioned in all reports, usually for early years classes but, in 2023, also used by older pupils to memorise number facts. A key difference in the 2023 report was emphasising more structured and teacher directed use of language through sentence stems, speaking frames, and choral responses.

What's the same and what's different?

We summarise the differences in Figure 1 - 'then' and Figure 2 - 'now'. Ofsted's previous view about the relationship between problem solving, calculation, and classroom talk is encapsulated in the aim that both primary and secondary pupils should 'learn to make sense of ideas, reason and justify their methods and solutions because discussion is a regular feature (UtS, 2008: 14).' Although other aspects of the guidance are distinct for different age phases, the aim of linking the three aspects is common to all.



Figure 1 Then: Ofsted's recommendations for problems, calculation and classroom talk

In contrast, CMS states that problem solving is different in primary and secondary. In primary the three aspects of mathematics teaching and learning are unconnected. The curriculum guidance is 'to apply facts and methods to wider problem-solving (CMS, 2023: 9)'. However, for secondary, all three aspects are linked, where teachers are told to 'make sure that pupils have sufficient opportunities to practise reasoning, explaining and problem-solving using the facts and methods they have been taught (CMS, 2023: 11).' Reasoning, explaining, and problem solving follow the teaching of facts and methods, and are things to practise; the emphasis is on them as something to know rather than something to do.



Problems



The paradigms

The flexible and restricted paradigms were apparent in the three foci. The flexible paradigm involved problem solving that was often open-ended and related to a variety of contexts, where pupils made choices about their approach and discussed their reasoning. The pupils had a range of calculation methods and were encouraged to select the most appropriate one for the question, justifying their choice. Classroom discussion was wide-ranging, multi-directional, and used to develop conceptual understanding. Flexibility was also apparent in other aspects of the report, such as pupil agency in choosing the starting level of their exercises and consulting pupils about their learning.

The restricted paradigm was represented by a more limited use of problem solving, viewing it primarily as word problems for practising calculations, taught through pupils copying models. Pupils are expected to use formal algorithms for all calculations. Classroom talk is primarily the teacher modelling and explaining, with primary pupils responding with choral responses and sentence stems. Even when older pupils reason, it is suggested that they replicate existing explanations and proof (CMS, 2023: 30). Pupil choice is limited because it risks teaching and learning deviating from the planned sequence.

View of	Flexible	Restricted	
Teachers and teaching	Pedagogue Responsive and adaptive Modelling mathematical thinking and enquiry	Technician Directed and directing Modelling methods	
Learners and learning	Capable of understanding different methods and applying in novel situations Agentic Choosing approaches Capable of being mathematical Experts in their own learning Independent Learning as understanding and relationship to mathematics	At risk of cognitive overload, need direction in novel situations and frequent testing Compliant Copying approaches Capable of learning in small steps Novices Dependent Learning as change in long term memory	
Mathematics and doing mathematics	Structure, patterns, processes and making connections Multiple approaches Problem solving as intrinsic and fundamental to mathematics Doing mathematics as reasoning	Factual, memorisable, concepts as relationships between facts Set methods Problem solving as an application or means of practising Doing mathematics as application	

Table 2 The flexible and restricted paradigms compared

Conclusion

The restricted mathematics paradigm limits what mathematics is and what learning mathematics can be. It is particularly egregious and potentially damaging when applied to early mathematics. Rather than, for example, seeing the exploration of mathematical concepts through objects, images and play as essential for the development of mathematical understanding, these are viewed in the recent Ofsted reports as stages or steps that should be passed over as quickly as possible.

We have aimed to show that Ofsted's current prescriptions depart from the consensus that existed between policy and mathematics educators in the past, which we have described as a flexible

paradigm. Some government-funded or supported guidance continues to reflect this flexible view. In the national curriculum and accompanying NCETM-developed guidance, this is exemplified in the way problem solving and the relationship between understanding and fluency are described. EEF guidance, informed by a rigorous synthesis of evidence of impact on learning, similarly echoes past recommendations. We contend the flexible paradigm is shared by many other bodies that currently provide curriculum schemes and professional development. This doesn't mean that the NCETM and other organisations haven't developed their thinking over time. They have. Particularly, about how they make what is known about rich mathematics teaching more commonplace in schools.

Ofsted's current view may relate to the application to mathematics of a generalised universal pedagogy of explicit instruction and a narrow conception of knowledge and learning as memorisation (RRM, 2021). Current differences between Ofsted and other influential bodies create challenges, not least for teachers and school leaders who must navigate conflicting advice. However, such differences gives ground for hope that the current turn by Ofsted could be a passing one and that dialogue continues between all who wish to foster mathematics teaching for learner independence and engagement.

*These schools were chosen as known examples of good practice. (Table 1)

References

Ernest, P. (1994) The Philosophy of Mathematics Education. Blackwell.

Ofsted (2008) Understanding the score: Messages from inspection evidence. www.stem.org.uk/resources/elibrary/resource/31194/mathematics-understanding-score

Ofsted (2011) Good practice in primary mathematics: Evidence from 20 successful schools. www.gov.uk/government/publications/good-practice-in-primary-mathematics-evidence-from-successfulschools

Ofsted (2012) Made to measure: Messages from inspection evidence. https://assets.publishing.service.gov.uk/media/5a8163dde5274a2e87dbd5f7/Mathematics_made_to_m easure.pdf

Ofsted (2021) Research review series: mathematics. A review of research into factors that influence the quality of mathematics education in schools in England. www.gov.uk/government/publications/research-review-series-mathematics

Ofsted (2023) Coordinating mathematics success. Mathematics subject report. www.gov.uk/government/publications/subject-report-series-maths/coordinating-mathematical-successthe-mathematics-subject-report

Nunes, T., Bryant, P. and Watson, A. (2008) *Key understandings in mathematics learning*. www.nuffieldfoundation.org/project/key-understandings-in-mathematics-learning