Diagrammatic representations for mathematical problem solving

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Introduction

Noble, Nemirovsky, Wright, and Tierney (2001) provide multiple references regarding the "strong support in the mathematics education community for the view that students should encounter mathematical concepts in multiple mathematical environments" (Noble et al., 2001, p. 85) and be able to connect these. This poster explores, via a case study, some of the related challenges for the restricted setting of using diagrammatic or graphical representations for arithmetic and algebraic problems. It uses a framing of 'lived-in spaces' (Nemirovsky, Tierney, & Wright, 1998) and the underlying research question is, "how can we support the use of diagrammatic or graphical representations becoming a 'lived-in space' for users?"

The case study is structured around a particular problem, set out as follows. There are three circular cardboard discs. A number is written on the top of each disc: (6), (7), (8). There is also a number (not necessarily the same) written on the reverse side of each disc. Throwing the discs in the air, and then adding the numbers on the faces, I have produced the following eight totals: 15, 16, 17, 18, 20, 21, 22, 23. Can you work out what numbers are written on the reverse side of each disc? (Association of Teachers of Mathematics (ATM), 1977).

In at least two instances, one – a professional development workshop with a group of 40 secondary mathematics teachers in England, and two – an online discussion group of mathematics educators, none of the initial shared approaches used a graphical representation of the problem, even when unknown variables were denoted by x, y and z, and could have suggested 3-D Cartesian space. This is striking as a graphical analysis of the problem can help bring to the fore much of the underlying structure. To clarify this, in our case study, two mathematicians, one who had solved the problem graphically and one who had solved it non-graphically, worked on it together for an hour. This was captured and analysed using multimodal microanalysis as by Nemirovsky and Smith (2013).

Theoretical Background

Those working on the above problem had access to graphical representations but what seemed absent is the creation of a *graphical space* (Nemirovsky et al., 1998) – a 'common place' where symbols and their referents are made accessible and sensible. Nemirovsky et al. (1998) develop three themes: *tool perspectives*, *fusion*, and *graphical spaces* to analyze students' use of a computer-based motion detector in the context of graphing. These three themes are not dependent on the technological nature of the tool and, we propose, apply equally to our setting, with graphical representation being the tool. Indeed, as Nemirovsky et al. (1998) observe, "Tool perspectives look at development of graphical space through simultaneously exploring the qualities of the tool and relation between actions and symbols. Fusion is about the blending of action and symbol in discourse within the graphical space" (Nemirovsky et al., 1998, p. 124). The growing familiarisation with a graphical space as it is populated with experiences and actions, which make it a space for purposeful and creative activity, is encapsulated by the notion of lived-in space. Noble et al. (2001) propose that "the mathematics that students learn from working in a given environment emerges from their process of making that environment into a lived-in space for themselves" (Noble et al., 2001, p. 86). Our work aims to draw out how the use of graphical representations can become a lived-in space.

Methodology and results

This case study uses a conversation between two mathematicians to investigate what actions and experiences contribute to fluid and effective approaches to solve the problem and how this can enable the fostering of a related lived-in space. Audio and video data were captured and analysed using constructs from the above referenced papers. The findings have two aspects: (i) an analysis of graphical and non-graphical solutions of the problem and (ii) observations on how the relevant lived-in space can be fostered, and related conclusions. We briefly outline the graphical approach:

1. The chosen number on a disc is independent of the other discs so choices can be modelled in 3-D space, one dimension for each disc. A choice of numbers, e.g., (6,7,8) gives a point in this space.

2. Flipping a disc results in a fixed addition or subtraction – representable by a translation vector.

3. It follows that there are eight choices of 3-tuples and these correspond to vertices of a cuboid.

4. The resulting constraints on the possible sums enable all possible solutions to be determined.

In relation to lived-in spaces, moving to-and-fro between different representations, translating expressions that are clear or articulable in one realm to the other, considering their affordances and constraints, and reflecting on these, provides opportunities for learners to make the graphical space a more familiar lived-in space. This also enables learners to experience and exercise graphical representation in ways which move it from being a representational tool to a more expressive tool.

References

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