#beingmathematical

Ashley Compton, Gerry McNally and Mary Pardoe discuss their experiences of joining in one of the regular #beingmathematical twitter discussions.

#beingmathematical is a one hour Twitter mathematical discussion organised by the ATM and open to all who are interested in mathematics education. Participants from around the world conquer time zone differences to join in and represent a wide range of ages taught, from EYFS to university. The writing opens with their thoughts on what they gain generally from #beingmathematical and is followed by their mathematical engagements with the task. Reflections on the intentions and rationale of the creator of #beingmathematical, Danny Brown, appear at the end of this piece.

Mary: I have enjoyed all the #beingmathematical events so far; making the small effort required to engage with a #beingmathematical task, and participate in discussion about it, is revealing and often surprising. Each #beingmathematical event provides an opportunity to think deeply about factors that may facilitate or inhibit mathematical learning. For example, the participants often approach, and reason about, the given task in different ways. They represent the 'objects-of-thought' differently, and so sometimes notice different properties and relationships, or notice the same relationships in different orders. It is delightful when we realise eventually that we have reached the same conclusion, which realisation was at first obscured by our individual 'ways-of-seeing' or our different, chosen modes of representation. Frequently I find those who work with concrete materials tend to ask themselves 'What would happen if …?' questions that are different to the extension-questions that occur to participants who have worked with drawn images. The implications of such findings are both fascinating and instructive.

Gerry: Since I first began to participate in the #beingmathematical sessions I've become hooked. Every session has helped me become better at learning, teaching and doing mathematics. This is due to the quality of the tasks themselves and, perhaps more importantly, to being part of an online community that shares my love of mathematics and my desire to engage actively and collectively with it; to exchange and discuss the diverse approaches, perspectives and findings that others bring to tackling and evaluating these unerringly rich tasks.

Twitter can be limiting as a platform for detailed or in-depth sharing. So, when I was offered the opportunity to contribute to a longer piece of writing on the May 9th session it didn't take me long to make up my mind to accept. By a fortunate coincidence, I had already decided to start a journal specifically to help focus my mind on #beingmathematical. This particular task is the first one for which I have a reasonably detailed and coherent record of my thoughts and actions as they arose and developed from when the task was first tweeted. My nascent journal is another consequence of the influence of people, predominantly ATM people, from whom I have begun to learn the value and develop the art of noticing what happens when I work on a mathematical task.

Ashley:I first joined in #beingmathematical in June 2018 and found it exhilarating but overwhelming, trying to follow the tweet stream while simultaneously working on the mathematics. Getting the problem a few days before the event was a huge improvement because it helped me to remember #beingmathematical was happening and it gave me a chance to investigate over time and reflect on my thinking. It's amazing how many of the tasks can be adapted very easily to make them appropriate for children in EYFS up to adult mathematics teachers.

One of the great things about #beingmathematical is that we are not afraid to say we don't understand someone else's work. We need to ensure that we establish this ethos in the classroom as well. During the #beingmathematical sessions we get to share our own approaches while trying to understand others' and make connections between them. The combination of independent work and group discussion is really powerful in leading to more insights into the mathematics. I wonder if a way forward would be to use tasks like this as homework activities so students can work at their own pace at home but then bring their work into class where the focus of the lesson is to work collaboratively, finding what's the same, what's different in the work and seeking patterns? It might result in a second piece of homework developing the ideas further.

The task for #beingmathematical on 09/05/19 was chosen by Julian Gilbey and is from the ATM publication *Functioning Mathematically* (see Figure 1).

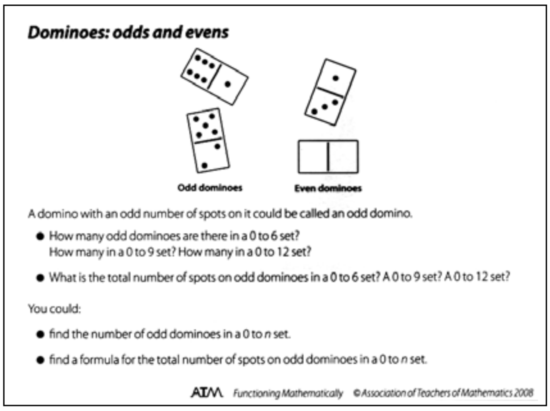

Figure 1: The task.

Mary's reflections

I worked on the task for an hour before the chat started. I first drew domino-sets ('double-6' to 'double-9') showing the total number of 'pips' on each domino, colouring those with an even number of pips red and those with an odd number of pips blue:

When I wrote numerical expressions for the total number of pips on the blue dominoes in the 'double-9' set, I began to think this exploration offered opportunities for older students to predict, conjecture and generalise. I began to see the possibility of students being challenged to write algebraic expressions to represent the generality of what they saw (that I was now starting to see). I decided to draw the 'double-10' set as well to help myself confirm, and get a clearer view of, patterns (see Figure 2).:

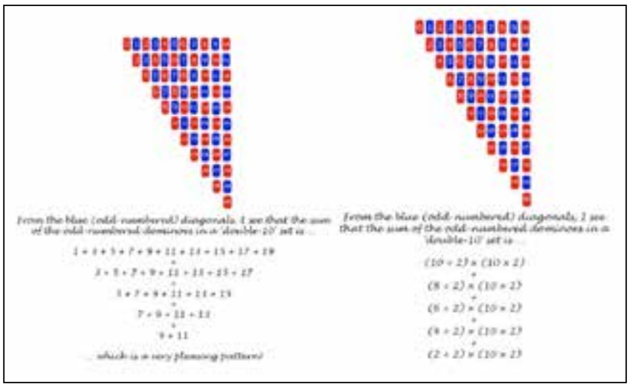

Figure 2: The double-10 set.

I set-about trying to simplify numerical expressions, predict results for sets of dominoes I hadn't yet drawn, and make conjectures about the general form of what I was seeing (see Figure 3):

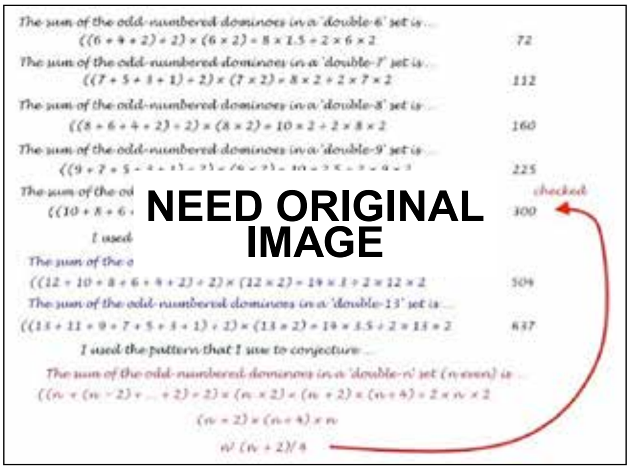

Figure 3: Conjectures and generalisations.

Doing this prompted me to begin to consider this 'task' as an opportunity for students to see the value of finding a simple way to express the sum of a finite linear sequence. Having done a little work on the 'situation-as-presented', having seen something of what other people had done, and having digested and responded to other people's tweets about their responses to the task, seeds of thought about an interesting and useful teaching idea had been sown.

Gerry's reflections

I started the task on my bus journey to work, the usual setting for my initial #beingmathematical play. This generally limits me to paper and pencil approaches and allows me to work in bursts of up to 40 minutes. It's an approach that also allows time for ideas to form, often on the periphery of attention, between periods of conscious effort. The task, as I recalled it from the previous evening's tweet was, "How many "odd" dominoes in sets of 0-6, 0-9, 0-12, 0-n?" Initially, then, I was only interested in whether and in how many cases the number of spots on a given domino was odd; I gave no regard to how many even dominoes there were or to the actual number of spots. I drew up a table for the standard 0–6 domino set (see Figure 4):

I immediately noticed how the horizontal totals (3, 3, 2, 2, 1, 1, 0, giving 12 overall) formed a sequence which I quickly came to think of as "counting down with duplication" (hereafter usually denoted by c-d-w-d). I wondered (and wrote in my notebook) if I would see a similar sequence for my next step, drawing up a 0-9 grid. I obtained the sequence 5, 4, 4, 3, 3, 2, 2, 1, 1, 0 (total 25) and noted the c-d-w-d pattern was only partly preserved, since 5 appeared just once. I wrote, "Is this because 9 is odd?". A further observation and question appear in my notes: "Highest number in c-d-w-d sequence is or is next to half of n [for a 0-n set]; for odd n, will this always round up?”

On a subsequent bus journey, I decided to draw up grids for n = 0, 1, 2, … and, this time, to include the totals of even dominoes, those whose spots total to an even number, for comparison. The grids for n = 2 to n = 6 are shown in Figure 4. Later, I wrote down some observations, highlighting two in particular:

- inclusion of even totals strengthens and extends pattern recognition
- completion of grids becomes algorithmic: grids are “triangular”, each line starting with E then alternating -O-E;
- counting-down-with-duplication patterns for “odd” dominoes are preserved, with the highest horizontal total being duplicated for even n but not odd n (the opposite is the case for “even” dominoes);
- **n/2 seems significant**
- **triangles seem significant**
- the c-d-w-d pattern can be thought of as two sets of triangular numbers (1 + 2 + 3 + …) merged together
- the total number of odds and evens combined for each 0-n set (which is, of course, just the total number of dominoes in the set) gives the (n+1)th triangular number (my totals for n = 0 to n = 6 were 1, 3, 6, 10, 15, 21, 28)
- The grids themselves formed a growing triangular pattern, each successive one being a copy of the one before with a new row added (or column, depending on the perspective you take).

I was ready to generalise my findings and wrote a title in my notebook: “Formulae for the numbers of: (i) odd dominoes; (ii) even dominoes, in a 0-n set.” When n is even (e.g. the 0-6 and 0-12 sets), the total number of “odd” dominoes can be obtained by adding together two “copies” of the triangular number corresponding to half of n. Symbolically, I can write this as

$D_{odd} = 2 \times T_{n/2}$

The situation for odd values of n (for example, the 0-9) set is more complicated: recall the earlier observations that the counting-down-with-duplication pattern is not fully preserved here. A little thought reveals we have two copies of the triangular number corresponding to half of (*n* – 1) and a single occurrence of the number is obtained by halving (*n* + 1). Hence,

$D_{odd} = 2 \times T_{(n-1)/2} + (n + 1)/2.$

By this time, I had a growing awareness that my thinking had become predominantly algebraic. Furthermore, I was drawing on my personal knowledge of triangular numbers and how these can be represented and evaluated by summation techniques, whilst considering and reflecting upon where and how this knowledge had arisen for me. As a secondary school mathematics teacher, I know only a tiny minority of my pupils would have access to such knowledge. I was reminded of the story about the young Carl Friedrich Gauss being able quickly to sum the numbers from 1-100, much to his teacher’s surprise, by duplicating the series, forming 100 pairs totalling 101, summing these (by multiplication) then halving the result. I carried on regardless, obtaining the following worked-up formulae:

$D_{odd} = n(n + 2)/4$, where n has even values and $D_{odd} = (n + 1)^2/4$ for odd values of n.

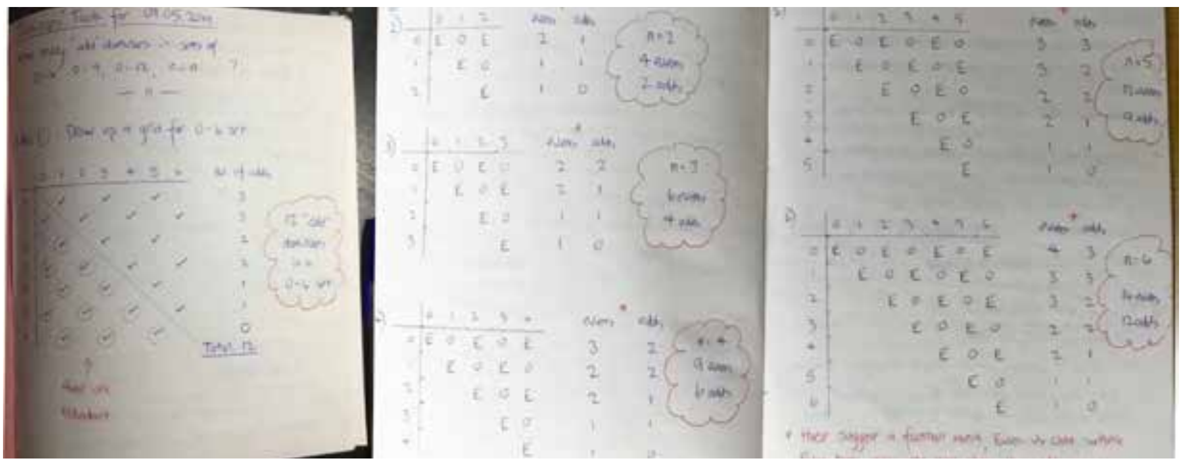

Figure 4: Gerry’s journal.

(Using a similar approach, I also obtained formulae for the number of even dominoes in a 0-n set for both odd and even n. The interested reader may wish to have a go at doing the same.)

I continued to explore the underpinning algebra in the lists I was creating. Although space precludes me from sharing all my findings here it is only in reconstructing my thoughts and actions for the purposes of contributing to this article that I realise how much I was caught up in the list of numbers itself rather than their origins and meaning in the context of the activity itself. This task is a powerful vehicle for facilitating mathematical thinking and behaviour. I endorse everything that has been said by my fellow participants regarding its pedagogical potential. The opportunities it provides to work systematically, record and analyse results, extract patterns and make and check predictions are invaluable. We had a ripe context for noticing and explicating simple (but hugely important) results like odd + odd = even, odd + even = odd, even + even = even, and expressing generalisations, whether in words or symbols. To be able to perform these mathematical acts collaboratively adds to their value to the mathematical development of the individuals involved.

At a more personal level, the act of recording my thoughts and actions in writing has helped me become more aware of my mathematical self. In writing this, some 4-5 weeks after the event, however, I have realised my notes are a scant record of what actually happened. There are ideas and actions I cannot account for and I truly do not know how long I spent on each episode or, indeed, how many episodes there were. I am coming to appreciate the importance of these acts of noticing and recording my own responses to mathematical tasks in enabling me to make sense of the mathematics itself.

Ashley's reflections

This task was particularly rich with a 'low threshold/ high ceiling'. There was a beautiful example of a class of 5-year-olds exploring the task, using Numicon to help identify the odd and even numbers. However, something accessible to these children was still challenging enough for a group of maths teachers. We could identify the odd and even numbers easily but we were challenged by trying to find a general formula for calculating the total number of spots on odd dominoes for both even (for example, ordinary 6 spot dominoes) and odd (for example, 9 spot dominoes) sets. This type of activity is ideal for mixed-attainment classrooms because it allows all children to engage but does not limit students who can go deeper into the mathematics.

Sorting the dominoes into odds and evens was the first step. Participants used different resources to explore this task and this led to different insights. Mary drew images of dominoes which she colour-coded for odds and evens; this highlighted diagonal patterns. She then replaced the spots on the domino with numbers showing the totals. I used physical dominoes which I separated into odd and even arrangements showing how many made each total (see Figure 5). This showed patterns in the different number of ways totals could be made. Gerry made a matrix on paper, which initially showed redundant combinations because the matrix had (0,1) and (1,0) as separate points whereas they are only one physical domino. Others used spreadsheets to make the matrix, putting the totals made into the cells, which highlighted a diagonal pattern of consecutive odd numbers.

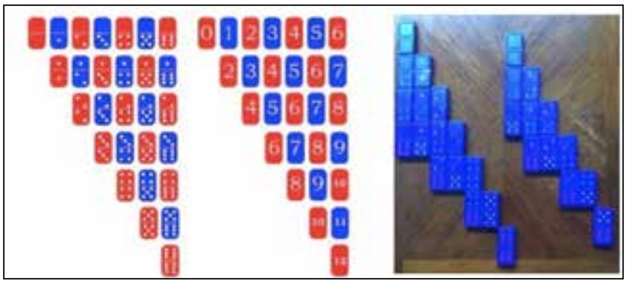

Figure 5: Drawing and using physical dominoes.

I used a physical set of dominoes and laid them out in a way I thought would help me see how many odd dominoes there were but also help with the next step of finding the total number of spots on the odd dominoes. Figure 5 shows the odd totals could be made in 1 way for 1, 2 ways for 3, 3 ways for 5, 3 ways for 7, 2 ways for 9 and 1 way for 11. This made a palindromic pattern of 1, 2, 3, 4, 3, 2, 1. The number of ways you could make the even totals was also palindromic: 1, 2, 3, 4, 3, 2, 1. This formation emphasised to me the pattern in the number of ways of making different totals and led me to see a relationship between the largest number of ways of making a total and the total number of spots; e.g. you can make 6 in 4 ways and the total number of spots on even dominoes is 96 which is 4 x 4 x 6, while for the odd dominoes you can make 7 in 3 ways and the total number of odd spots is 72, which is 3 x 4 x 6. I surmised that the 6 related to using 0 - 6 spot dominoes. This was confirmed when the 0-9 spot dominoes had totals made by 5x5x9 and 5x6x9 and the 0-12 totals were 6x7x12 and 7x7x12.

I spent 3 to 4 hours playing with the dominoes, numbers and patterns before the session. My first session was about 2½ hours and was only interrupted by the need to cook dinner. I then came back to it the next day and was able to reflect on what I had done and try to turn the patterns I had seen into formulas. Is this practical in the classroom where lessons are often short? Will a class of students have this level of interest and perseverance? Some participants gave examples of their students staying focused on investigations but I was remembering a recent tweet from a teacher on maths CPD who was complaining about maths teachers assuming that everyone would be as interested in mathematics problems as they were. As someone who gets excited by such problems I need to bear this in mind.

Final thoughts: Danny Brown

I wanted, through ATM, to shift the conversation on Twitter from *talking about* mathematics and mathematical pedagogy, to the mathematics itself. It seemed to me many people on Twitter liked to *talk about* mathematics, with all of the miscommunications, myths, and divisions this perpetuates. I was guilty of contributing to these so-called debates over the years, and realised the fruitlessness of *talking about* in the absence of shared personal experience. #beingmathematical, then, was about bringing people back to *shared mathematical experience*, doing mathematics together in order to see what we could glean about pedagogy. This is nothing new and has been happening at ATM conferences and branch meetings for decades.

I feel there are limitations to #beingmathematical. I personally wanted us to do the mathematics live, as it is all about the *process* not the product for me - I always wanted to know what people were doing as they were doing it, rather than hear the narrative they tell afterwards. Of course, it would take a great deal of mathematical security to expose one›s vulnerability on Twitter. And it is hard to communicate mathematical ideas on Twitter - for me it is preferable to do mathematics in the flesh - but it does mean, along with the limitations to message size, participants have to find ways ingenious ways to communicate their ideas.

What Twitter also offers is the opportunity for people of diverse experience and location to come together. I am very proud of what #beingmathematical has developed into - the sense of community is developing - although I was always disappointed each week that more people didn't get involved, particularly those who are very happy to *talk about* mathematics education to their thousands of followers, and who I felt could contribute a great deal by participating in this community of people doing mathematics together and sharing their experience.

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Mary Pardoe is a former Secondary teacher and mathematics adviser.